is briefly discussed (p. 296–297) in the chapter on special functions. Additional numerical data also include exact values of the first 17 Bernoulli numbers and the first 10 Euler numbers, 10D approximations to $\zeta(n)$ for $n = 2(1)11$, and Euler's constant and Catalan's constant to 16D and 9D, respectively. I have examined all these data carefully, and the errors detected, together with errors in the formulas, are enumerated separately in this issue (MTE 293).

Use of the book is facilitated by an elaborate index of special functions and notations on p. 417–422. In addition to supplementary remarks and the bibliographies already mentioned, the Appendix contains (on p. 423–429) a discussion of the variations in the notation and symbols used for special numbers and functions throughout the mathematical literature and a concise list of abbreviations (p. 432–433).

The lucid expository style employed throughout is exemplified in the Introduction. Here, a systematic summary of definitions and theorems relating to infinite products and infinite series of various types supplements the list of relevant formulas. Similar explanatory text serves as introduction to several of the subsequent chapters and their subdivisions.

Typographical errors found in the text are minor and do not detract from the intelligibility of the textual material. The typography, especially in a compilation of such a large number of formulas, is uniformly excellent, and the appearance of the book is attractive. Professor Archibald's opinion that the first edition was "undoubtedly of considerable value for any mathematician to have at hand" certainly holds true for this latest version.

J. W. W.

1. I. M. Ryzhik & I. S. Gradshteĭn, *Tablitsy Integralov, Summ, Riadov i Proizvedeniĭ*, [Tables of Integrals, Sums, Series and Products], The State Publishing House for Technical and Theoretical Literature, Moscow, 1951.
2. R. C. Archibald, **RMT** 219, *MTAC*, v. 1, 1943/45, p. 442.
3. Bierens de Haan, *Nouvelles Tables d'Intégrales Définies*, Leyden 1867. Reprinted by G. E. Stechert & Co., New York, 1939.

70[G].—Eugene Prange, *An Algorism for Factoring $X^n - 1$ over a Finite Field*, AFCRC-TN-59-775, U. S. Air Force, Bedford, Mass., October 1959, iii + 20 p., 27 cm.

An algorism is given for factoring $X^n - 1$ over the finite field F_q of q elements. This can be of use in constructing another finite field over F_q, in constructing a linear recursion of period n over F_q, or in constructing cyclic error-correcting group codes. The algorism has two parts: Step 1, the construction of the multiplicative identities of the minimal ideals of $F_q[X]/[X^n - 1]$; Step 2, the use of these idempotents in the construction of the irreducible factors of $X^n - 1$.

Author's Abstract

71[G].—M. Rotenberg, R. Bivins, N. Metropolis & J. K. Wooten, Jr., *The 3-j and 6-j Symbols*, The Technology Press, Massachusetts Institute of Technology, Cambridge, Massachusetts, 1960, viii + 498 p., 29 cm. Price $16.00.

Wigner's 3-j symbol is closely related to the Clebsch-Gordan coefficients used in the coupling of angular momenta. If $\mathbf{J}_1$ and $\mathbf{J}_2$ are coupled to give $\mathbf{J}$, with j, j_1, j_2 as the total-angular-momentum quantum numbers and m, m_1, m_2 as the quantum numbers for the z-components, the expansion coefficient giving the coupled states in terms of the uncoupled are

$$(j_1 j_2 jm \mid j_1 m_1 j_2 m_2) = (-1)^{j_2 - j_1 - m}(2j+1)^{1/2} \begin{pmatrix} j_1 & j_2 & j \\ m_1 & m_2 & -m \end{pmatrix}.$$

Here the symbol on the left is the expansion coefficient in the notation of Condon and Shortley, *Theory of Atomic Spectra*; the last symbol on the right is the Wigner 3-j symbol. The advantage of a tabulation of the 3-j symbols,

$$\begin{pmatrix} j_1 & j_2 & j_3 \\ m_1 & m_2 & m_3 \end{pmatrix},$$

rather than of expansion coefficients results from the high degree of symmetry of the 3-j symbols. At most a sign change results from an interchange of columns or from changing the signs of all the m's. Thus, from these tables, which are restricted to

$$j_1 \geqq j_2 \geqq j_3 \quad \text{and} \quad m_2 \leqq 0,$$

all expansion coefficients can be obtained for any $j = 0, \frac{1}{2}, 1, \frac{3}{2}, \cdots, 8$.

The 6-j symbols occur in the coupling of three angular momenta $\mathbf{J}_1$, $\mathbf{J}_2$, and $\mathbf{J}_3$. One can either couple $\mathbf{J}_1$ and $\mathbf{J}_2$ first to obtain $\mathbf{J}'$, and then couple $\mathbf{J}'$ to $\mathbf{J}_3$ to obtain $\mathbf{J}$—this coupling scheme results in quantum numbers j_1, j_2, j', j_3, j, m—or one can first couple $\mathbf{J}_2$ and $\mathbf{J}_3$ to obtain $\mathbf{J}''$, and then couple $\mathbf{J}''$ to $\mathbf{J}_1$ to obtain $\mathbf{J}$—this scheme results in quantum numbers j_2, j_3, j'', j_1, j, m. The overlap integral between these two representations is given by

$$(j_1 j_2 j' j_3 jm \mid j_2 j_3 j'' j_1 jm) = (-1)^{j_1 + j_2 + j_3 + j}[(2j'+1)(2j''+1)]^{1/2} \begin{Bmatrix} j_1 & j_2 & j' \\ j_3 & j & j'' \end{Bmatrix},$$

where the last symbol is a 6-j symbol. The 6-j symbol tabulated,

$$\begin{Bmatrix} j_1 & j_2 & j_3 \\ l_1 & l_2 & l_3 \end{Bmatrix},$$

has sufficient symmetry that it need be listed only for

$$j_1 \geqq j_2 \geqq j_3, \quad j_1 \geqq l_1, \quad j_2 \geqq l_2, \quad j_2 \geqq l_3.$$

Within these restrictions, it is listed for all half-integral values of the j's and l's from 0 to 8.

The tables were computed on the MANIAC II at the Los Alamos Scientific Laboratory. An adequate 40-page introduction describes the symbols and their uses. Since the symbols are the square-roots of rational fractions, the squares are tabulated as powers of primes in a shorthand notation, with an asterisk used to denote the negative square root. For example, the entry $*\underline{1}5\underline{1}0,\underline{2}\underline{2}\underline{2}1$ is to be interpreted as

$$-\left[\frac{3^5 \times 7^0 \times 19^1}{2^1 \times 5^1 \times 11^2 \times 13^2 \times 17^2}\right]^{1/2}$$

George Shortley

Booz-Allen Applied Research, Inc.
Bethesda 14, Maryland